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Hedging investors' exposure to the Greek banking system with index futures

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ABSTRACT

In this dissertation we study the effectiveness of a hedging strategy with futures. We consider the case of a risk-averse investor that maintains a long position on a weighted portfolio of bank stocks and we use index futures as a hedging instrument. Our main focus is on the period of extreme financial crisis for the Greek stock market that highly affected the Banking sector. Results to a static minimum-variance hedging approach reveal that a hedging portfolio with index futures can eliminate the 80% of the total variation of the banking exposure. The simple hedging strategy almost always outperforms the cumulative returns of a buy-and-hold strategy on the banking index. Only at the last several months of our sample the effectiveness of the hedge is slightly decreased, but this is due to the higher basis risk that was caused by the declines in the correlations between the banking sector and the market returns.

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Introduction

There is no counterargument that the global economy has been facing a period of extreme financial instability during the last several years. The crisis has become contagious across countries and this is mainly due to the rapid expansion of the traded volume of financial derivatives that actually "boosted" the banking sector. Since banks hold positions of very high risk in their portfolios, this risk is translated into systematic risk for the total economy, and this is because banks have become big enough to drive financial markets.

This has also been the case in Greece, where we see that the Greek Banking sector comprises the biggest component of the major Greek stock market index. We could think of Greece as an emerging market, but this was not in terms of heavy industries but mostly in terms of services, including accounting and financial sectors. Before the financial crisis we experienced a period of extreme debt positions due to the easy credit environment. Greek banks were continuously lending money, securing their positions with derivative securities. This policy led to extremely increased leverage for the banks, which made them extremely vulnerable to financial instabilities. Just like the US, these conditions allowed for housing speculation and led to a bubble.

When the housing bubble started deflating after 2007, the households and bankrupt companies were not eligible to repay the debt to the banks. The banks tried to collect and sell assets in order to maintain their liquidity to a sustainable level but this was not easy anymore, thus leading to an unprecedented "liquidity shortfall". Interest rates began to increase significantly, and the value of the assets decreased significantly since they were not easily liquefiable. The banks had invested in MSB (Mortgage backed securities), CDOs (Credit default obligations) and other credit derivatives that also started producing significant losses. This global contagion that was first triggered by the housing bubble in the US was reason for many financial institutions to collapse and many banks to bailout. The Greek banking sector also started shrinking rapidly and the total Greek stock market experienced extreme downturns due to these unexpected losses.

The aim of this dissertation will be to study the hedging performance of an investor that maintains an exposure to an instable banking system. It is very challenging to study the case of the Greek banking sector, especially during the recent period of the financial crisis. The Greek stock market experienced a period of very high volatility accompanied by extreme losses and as we explained in the above analysis, these losses were due to the systematic economic shocks that were transmitted into the banking system.

In order to be more specific we will try to show how using index futures contracts would allow an investor to hedge his exposure to the Greek banking sector. We aim to study the hedging performance of a simple minimum-variance hedging strategy for the banking index. We will consider an investor that has a long position into a portfolio of bank stocks and at the same time maintains a short position into stock index futures that are expected to expire soon. Since for the case of Greece the banking sector strongly interacts with the total market index, we expect this hedging strategy to produce sufficiently good results. Our primary interest is to provide an economic interpretation of the results in the sense of explaining how important a hedging would be for a Greek investor during the recent financial crisis.

Results from a linear static estimation reveal that an investor can hedge himself from a significant proportion of the total price variation of the banking index by shorting an optimal amount of index futures. The optimal hedge ratio turns out to be 1.34 units of index futures per unit of the asset that is to be hedged, in this way providing a hedging effectiveness of 80% for the investor. Especially during the downward movements of the market, the hedging portfolio significantly outperforms the buy-and-hold strategy on the Banking index. The returns to the hedging portfolio are much less volatile and close to zero, again showing the effectiveness of the hedging strategy. There is an increase in the volatility of the hedged portfolio returns during the last several months of our sample, but this is explained by the higher volatility of the banking index during that period. The FTSE-B and the FTSE futures started becoming less correlated, hence introducing a higher basis risk in the hedging strategy and making the imperfectness of the hedge slightly more pronounced.

This dissertation is organized in the following way. In the next section we present a detailed literature review on the optimal hedge ratios for futures. Then we describe the data that we use and the methodology in order to implement the hedging strategy. We then explain our findings regarding the hedging performance of stock index futures for an investor that was exposed to the Greek banking sector during the financial crisis.

Literature Review

The debate on mean-variance hedging with futures started with the seminal paper of Johnson (1960). In his paper he outlines and appraises the theory of "hedging and speculation" that is based on the minimum variance criterion by applying it on commodity futures contracts. His work was followed by that of Ederington (1979), who describes the hedging procedure with GNMA futures (Government National Mortgage Association) and introduces a measure of effectiveness of the hedge. Some years later, Figlewski (1984) was the first to focus on the hedging effectiveness of stock index futures (using the S&P500 index) and to analyze the components of basis risk.

The minimum-variance hedging procedure has been widely used from both academics and practitioners.[1] Yet, there is a strand of literature that casts doubts on whether this is an appropriate hedging technique, suggesting other objective criteria in order to implement the hedge. Cecchetti et al. (1988) go beyond the minimization of the variance of the portfolio in order to take into consideration the expectations on the mean return and the time-variation of the future cash flows. In their paper, Howard & D'Antonio (1984), try minimizing a risk-adjusted measure (Sharpe ratio) instead of minimizing variance. Other papers like that of Cheung et al. (1990) or Lien & Luo (1993) and Lien & Shaffer (1999) estimate the optimal hedge ratios and assess the effectiveness of their strategies by optimizing the mean-Gini coefficient. In order to achieve a better hedging for downside risk, Eftekhari (1998) as well as Lien & Tse (1998) and Lien & Tse (2000) change their objective criterion to minimizing lower partial moments.

The most straightforward and easy method for estimating hedge ratios has been the simple linear regression. But more recently, advanced econometric methods have been used not only in order to provide a more accurate estimation of the hedge ratios, but also to capture their dynamic time-varying behavior. Baillie & Myers (1991) note that the standard static approach to estimating the hedge ratios is not appropriate and they suggest that a bivariate GARCH estimation can produce superior results, thus taking into account the time-varying

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[1] A very nice review on the futures hedge ratios can also be found in Chen et al. (2003).

nature of the hedge. Moschini & Myers (2002) also use a GARCH parameterization in order to capture the time-variability of volatility for testing more accurately whether the hedge ratios are time-invariant. Chan & Young (2006) consider a jump component with autoregressive intensity in the bivariate GARCH model and conclude that there is significant information lying within the jump dynamics that can improve the time-varying effectiveness of the hedge. Lee & Yoder (2007) use a more advanced bivariate Markov regime-switching BEKK-GARCH model, that is also produces superior estimates to the time-varying hedge ratios. Generally, there are numerous applications that use much more complicated methods of estimating the hedge ratios, which underlines the importance of achieving accuracy in the results when building a hedging position in practice.

The aforementioned papers and techniques have mainly focused on commodity markets in order to examine and improve the effectiveness of the hedge. But these methods can also be applied to other instruments such as stocks, indices or even fixed income securities. For example, Koutmos & Pericli (1999) as well as Bhattacharya et al. (2006) show that a dynamic error-correction GARCH model can provide superior results than a static hedge when applied to GNMA MBSs (Mortgage-Backed Securities). In all circumstances there are several limitations regarding the effectiveness of the estimated hedge ratios that arise due to the variability of the basis risk or in other cases the illiquidity of the derivative securities that are used to implement the hedge.

However, the majority of the hedging applications on the minimum variance hedge ratios focuses on stock market indices. This task is also challenging since the correlations among financial assets in the stock market vary significantly, hence introducing a strain in achieving a perfect hedge. As we have already mentioned, Figlewski (1984) started analyzing the hedging effectiveness with S&P500 index futures, at the same time discovering the driving factors of basis risk. Several papers followed trying to investigate the impact of other portfolio characteristics on the hedge ratio estimations. For instance, Jennings & Graham (1987) find that equity portfolios with higher dividend yield ratios produce more effective hedges, especially for short-term hedging horizons. Geppert (1995) also studies the effect of the investment period length on a minimum-variance hedged portfolio. Lindahl (1992) examines the impact not only of the duration of the hedge but also of the contract expiration, but he finds relatively mixed results compared to a naïve hedging strategy. Merrick (1988) points out that there is a significant impact on the optimal hedge ratios that is due to a possible mispricing of the futures returns.

As above, advances in econometric methods and estimation procedures have allowed academics to study the time-variability of the hedge ratios, not just for hedging commodities, but for hedging stock market positions as well. Myers (1991) finds that the GARCH representation can lead to a superior hedging performance. Lien et al. (2002) compare the rolling hedge ratios generated from a constant-correlation V-GARCH model to those of a simple OLS method but find no significant improvement. On the other hand, Poomimars et al. (2003) conclude that dynamic estimations of the optimal hedge ratios lead to improvements in the performance of a hedge. Harris & Shen (2003) also capture time-varying hedge ratios by using rolling regressions or GARCH and EWMA models. When comparing the OOS performance of time-varying hedge ratios to naïve OLS estimates, Choudhry (2003; 2004) finds that GARCH models can produce results that outperform those of static ratios. In addition to his results, Miffre (2004) states that in many cases the conditional rolling OLS estimates of the hedge ratios are superior to those of an unconditional model. The findings of Yang & Allen (2005) are also in line with these papers. Alizadeh & Nomikos (2004) state that MRS (Markov Regime-Switching) models can provide even better results than the GARCH.

However, from a practical point of view it would be quite costly to update the hedging portfolio very often in order to account for the time-variation of the hedge ratio, since the transaction costs would erode any profits. Hence, from a long-term perspective, it would also be interesting to examine if there is a cointegrating relation between spot and future prices Ghosh (1993). These studies suggest that if there is a long-run equilibrium between spot and future prices, then an error-correction mechanism should drive any short-run deviations back to this equilibrium. There are many other papers that try to compare the performance of error-correction models to other approaches for estimating the hedge ratios (see also Ghosh (1993), Kroner & Sultan (1993), Park & Switzer (1995), Chou et al. (1996), Ghosh & Clayton (1996), Tong (1996), Choudhry (2003), Alizadeh & Nomikos (2004) and others). Others, like Garbade & Silber (1983) or tried to capture the impact of a lead-lag relationship between the spot and future prices. Kroner & Sultan (1993) and Miffre (2004) tried to incorporate conditional information in their approaches as well. Generally, there is some small improvement by using cointegration techniques, but according to Lien (2004) the loss of hedging effectiveness by omitting a cointegration relationship is only minimal. Tong (1996) also supports this statement.

There are numerous papers that aim to demonstrate the superiority of more complex dynamic models in estimating hedge ratios compared to other more simple approaches. In their analysis, Park & Switzer (1995) use a symmetric bivariate GARCH model in order to show that dynamic hedge ratios are superior to static hedge ratios, and this is the case for many different indices. Tong (1996) also provides evidence in favor of the dynamic GARCH model compared to simple OLS and cointegration approaches. Since there is evidence of asymmetric responses of stock market volatility to good and bad news, Brooks et al. (2002) support that it is important to allow for time-varying as well as asymmetric optimal hedge ratios. Choudhry (2003) also compares different approaches and finds that the GARCH model gives better results. Alizadeh & Nomikos (2004) use a Markov-Switching GARCH model and they compare it not only with the classical GARCH, but also with the OLS and error-correction models. Dark (2004) introduces a bivariate error-correction FIGARCH model in order to capture time-varying correlations and compare its performance to that of a bivariate error-correction GARCH and a static naïve OLS estimation of the mean-variance hedge ratio. Yang & Allen (2005) find similar results. Floros & Vougas (2004) focus on the Greek stock futures market using data from the Athens Derivatives Exchange (ADEX) and they conclude that the multivariate GARCH approach is superior to others. Laws & Thompson (2005) estimate an EWMA model as well and find that in their sample the EWMA approach gives improved results in terms of hedging effectiveness. Alexander & Barbosa (2007) provide further evidence in favor of dynamic hedge-ratios when time-varying conditional variance-covariance matrices are taken into consideration. Lai et al. (2009) construct a more advanced copula-threshold-GARCH technique. Their findings show that a Gaussian or a Mixture Copula model give hedge-ratios that perform quite good in terms of reducing the variance of a hedging portfolio and producing higher returns.

Generally, there is no consensus regarding the most efficient method to estimate hedge ratios. The efficiency of the hedge improves only if correlations between the prices of the unhedged position and the hedging instrument are high. Thus, there are many scholars that cast doubts on the necessity of advanced econometric techniques in order to estimate the hedge ratios.

Moosa (2003), Lien (2005), Poomimars et al. (2003) and many others conclude that the majority of the econometric applications produce similar results and there is only a minor improvement by using advanced and complicated models.

Minimum-Variance Hedging

Concept of Hedging

Let us consider an individual investor that wishes to use futures contracts to hedge his position. His target is to minimize the total risk of his portfolio in order to neutralize his exposure to risk. If the investor maintains a long position in a stock (or portfolio) he will need a short futures position in order to offset the risk and vice versa.

For instance, consider a UK company that expects to receive 500,000 € on the 1st of December for goods that have already been sold 3 months earlier. Lets say that the £/€ exchange rate on the 1st of September was 0.704£/€. If the company did not hedge its position, it would be exposed to currency risk in case the spot £/€ rate falls. Suppose the £/€ futures rate for 3-months were 0.7047£/€. If each futures contract was denominated in 100,000€, then in order to hedge this position the company needed to short five 3-month Euro-futures. In that case it would receive 500,000€ from the creditor at the 1st of December and receive £352,350 under the futures contract by selling the Euros at the fixed rate 0.7047£/€, thus avoiding the currency risk from a possible devaluation of the Euro.

Basis Risk

Although it is very useful to hedge a position, the example presented in the previous section is not very realistic. Since futures contracts are standardized a hedger may not be able to eliminate risk totally and this is due to many reasons:

- First of all, an investor may not know will complete certainty the exact date of the payoff of his assets.
- Secondly, the underlying asset of the futures contract may not be exactly the same as the asset that one wants to hedge.
- Finally, the hedge may lead to a mismatch of expiry dates and in that case the hedger may need to close out the position to futures.

These differences can lead to imperfect and varying correlations between changes in the price of the hedging instrument and the underlying asset that is to be hedged and hence introduce the so-called "*basis risk*".

The *basis* in a hedge is defined as:

$$B_t = F_t - S_t$$

where S_t is the spot price of the asset that is to be hedged and F_t is the price of the futures contract. The basis can be either positive or negative and it is time varying since it depends on prices. Usually it is smaller for financial assets that are more liquid than for commodities and it is more variable in the cases where the hedging instrument is not very correlated with the asset that is to be hedged.

The selection of the futures contract that will be used to hedge a position is a quite important decision if one wants to achieve a good hedging performance. Futures must be selected in such a way that the basis risk is minimized; in other words the correlation between the instrument and the position is maximized. This means that the nature of the *underlying asset* of the futures contract must be similar in order to co-move with the spot price of the financial asset that is to be hedged. Moreover, the selection of the *expiry date* of a contract is also crucial. One has to select a hedging instrument that does not expire before the preferred date. Normally a hedger selects the first contract that is about to expire just after the horizon of his position. After the period of his investment has passed, he can close out his position in futures contracts before they expire by taking the opposite position in futures.

Calculation of the Minimum-Variance Hedge Ratio

A perfect hedge is only possible if the spot and the futures prices of the asset are perfectly correlated, which means that $|\rho| = 1$. This is only possible in the ideal case where the underlying asset of the futures contract is the same as the asset that is to be hedged and the horizon of the hedge is exactly the same as the expiration of the futures contract. In this case the basis risk is actually zero at the end of the hedging horizon. This practically means that the volatility of the payoffs of a portfolio containing both the asset and the futures is zero because the variance of the instrument perfectly offsets the variance of the asset.

However, as we have already mentioned, this is not possible in real-life since correlations among different financial assets are imperfect, $|\rho| < 1$. It is not easy to perfectly match the same underlying asset and the expiration date in practice. This imperfect hedging introduces variance in the portfolio of the hedger, who remains exposed to some extent to the changes of the basis, i.e. the basis risk.

Let us consider an investor with a natural long position in an underlying asset that can be hedged with an opposite short position in futures. Since he might be exposed to basis risk, he wants to optimize the performance of his hedge by estimating the optimal hedge ratio that minimizes the risk of his portfolio, i.e. the variance of the payoffs of the hedged position. Practically, he wants to calculate the number of futures contracts that he needs to short.

The value π of his portfolio at time t is:

$$\begin{aligned}\pi &= QS_t - H(F_t - F)\\ &= QS + Q(S_t - S) - H(F_t - F)\\ &= QS + Q\Delta S - H\Delta F\end{aligned}$$

where Q is the size of the position to be hedged, S is the current spot price of the asset, S_t is the uncertain future spot price of the asset at the target date t, H is the size of the position in futures, F is the current futures price that expires at time T and F_t is the uncertain price of the same futures contract at time t. At the equation above $\Delta S = S_t - S$ is to denote the change in the spot price of the asset and $\Delta F = F_t - F$ is the change in the futures price that is used as a hedging instrument between the time of the set-up and the target date t.

Generally, one can define the **hedge ratio** to be $h = H/Q$, which is the number of units of futures held per unit of the underlying asset. This means that the portfolio value can be written as:

$$\pi = Q(S + \Delta S - h\Delta F)$$

As we have already said, the **optimal hedge ratio $\boldsymbol{h}^*$** is the hedge ratio that *minimizes the variance of the payoffs π of the hedging portfolio*. This means that the number N of futures contracts that should be shorted is:

$$N = \frac{H^*}{Q_F} = h^* \times \frac{Q}{Q_F}$$

where Q_F is the size of each futures contract and H^* is the optimal size of the position that needs to be taken in futures contracts.

The variance of the payoffs of the portfolio that needs to be minimized can also be written analytically in the following way:

$$var(\pi) = Q^2(\sigma_S^2 + h^2\sigma_F^2 - 2h\sigma_{S,F})$$

where σ_S^2 is the variance of the change in the spot price of the asset ΔS and σ_F^2 is the variance of the changes of the futures prices ΔF. The notation $\sigma_{S,F}$ refers to the covariance between ΔS and ΔF.

In order to minimize the above objective function, one needs to apply the first order conditions, that is the first derivative of the variance function with respect to the hedge ratio:

$$\frac{\partial var(\pi)}{\partial h} = 0$$

By solving this equation one can derive the optimal hedge ratio h^*:

$$h^* = \frac{\sigma_{S,F}}{\sigma_F^2} = \rho \frac{\sigma_S}{\sigma_F}$$

This means that at the optimal point, the minimized variance of the portfolio $var(\pi^*)$ equals:

$$\begin{aligned} var(\pi^*) &= Q^2\left(\sigma_S^2 + {h^*}^2\sigma_F^2 - 2h^*\sigma_{S,F}\right) \\ &= Q^2\sigma_S^2(1-\rho^2) \end{aligned}$$

The effectiveness of the hedge is the proportion of the variance that is eliminated by hedging. Theoretically the effectiveness of a hedging strategy can be measured by ρ^2; (the actual effectiveness can be smaller in reality due to rounding, since h^* may not be an integer).

Econometric method for estimating the optimal-hedge ratio

A closer look at the analysis above reveals that the most straightforward and intuitive methodology to estimate the optimal hedge ratio is actually a simple linear regression. In order to demonstrate the connection between the theory and practice let us first consider a linear model of the form $Y_t = a + \beta X_t + \epsilon_t$. It is known from econometrics that the OLS (ordinary least squares) estimator is the one that minimizes the sum of the squared residuals. This means that the slope coefficient equals:

$$\beta = \frac{cov(X,Y)}{var(X)} = \frac{\sigma_{X,Y}}{\sigma_X^2} = \rho_{Y,X}\frac{\sigma_Y}{\sigma_X}$$

since the correlation coefficient is $\rho_{X,Y} = \frac{\sigma_{X,Y}}{\sigma_X \sigma_Y}$. Similarly, the optimal hedge ratio is estimated in the following way:

$$h^* = \rho\frac{\sigma_S}{\sigma_F}$$

which is the slope coefficient β of a linear regression of the changes in the futures prices versus the spot prices of the asset. This means that we have a linear regression model of the form:

$$\Delta(S_t) = a + \beta\Delta(F_t) + \epsilon_t$$

After computing the optimal hedge ratio for our data series we will try to see how effectively the hedging strategy behaves. In order to accommodate this issue we will focus on the ratio that measures the effectiveness of the hedge, that is estimated as:

$$\rho^2 = h^{*2}\frac{\sigma_F^2}{\sigma_S^2}$$

which actually equals the squared correlation coefficient between the spot and the futures prices. From a statistics point of view this can be directly estimated by the coefficient of determination R^2 of a linear regression with an intercept term, which is the percentage of the variation of the dependent variable that can be explained by the variation of the regressor. From our perspective it measures the hedging effectiveness since it captures the variation of the spot returns of the natural position to the asset that is offset by the variation of the changes in the futures prices.

This estimation procedure, also described in (Figlewski 1984) and (Hull 2009) with more details, is the most widely used procedure in the literature because it is easy to apply and interpret. As we have shown in the literature review, there are many other econometric procedures that can be used to estimate the optimal hedge ratio. The most direct alternative to the simple linear regression is the cointegration analysis. Financial time series are usually found to be nonstationary in their levels, i.e. raw price data normally have a unit root, but their first differences are stationary. In other words prices are integrated of first order, i.e. $I(1)$ since the returns are stationary. Hence one can apply an error-correction model on prices and see whether there is a long-run trend between the spot and futures prices.

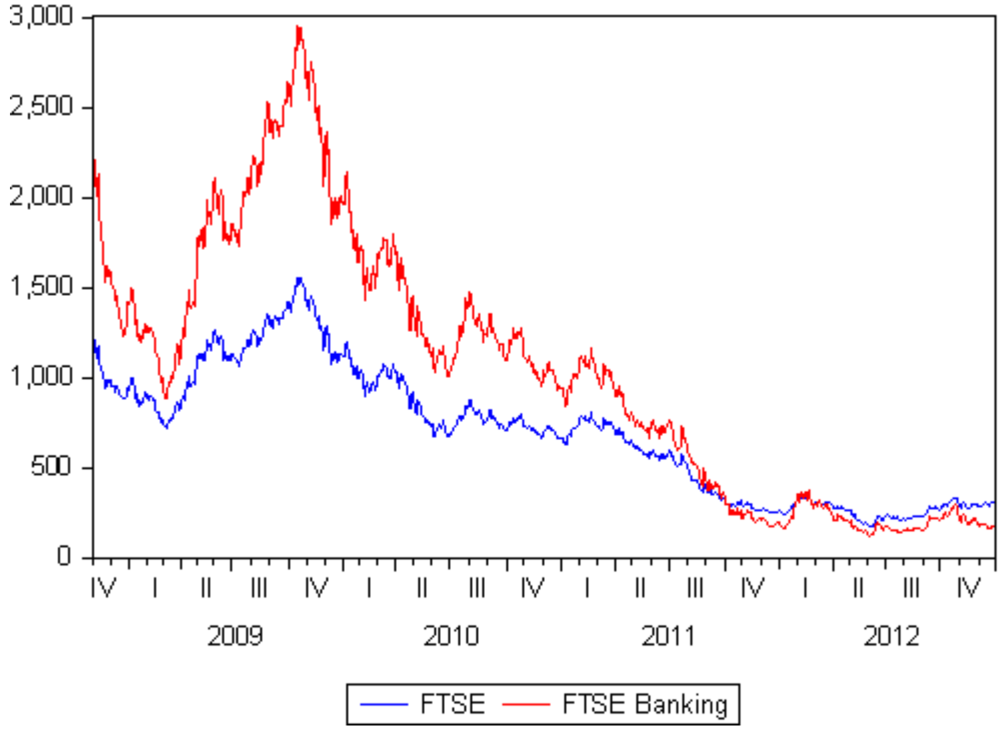
Spot & Futures Data

Our main purpose in this dissertation is to study the performance of a hedging strategy for a Greek investor that was exposed to the Greek Banking sector during the recent financials crisis. This means that in our analysis we will consider an investor that has a long position in the Greek banks. The best proxy will be to use the FTSE-B index from the Athens Stock Exchange (ASE), which is actually a weighed portfolio of all the Greek Banks. Thus, we collected data of daily closing prices for the major Greek large-cap index FTSE/ASE20 and the Banking index FTSE-B. The data are collected from the official database of the Athens Stock Exchange (ASE) and cover 4 years of data, approximately from 2009 to 2012. To be more specific, we start from the date that the FTSE-B index was actually introduced, that is 31/10/2008, until the end of 2012, which gives us 1042 observations in total.

As a hedging instrument we will use the FTSE main index futures since they are very liquid and better priced. Besides we believe that this will be a very efficient hedge because the Greek banks are the most important component of the Greek stock market and hence the correlations between the stock prices of the Greek banks and the market (i.e. the FTSE20 index) will generally be very high. Note that we will only be using short-term futures (i.e. the futures on the index that are about to expire very shortly) since these futures are more liquid (compared to other futures with longer expiration horizons) due to the higher traded volume and thus they carry more information in their prices. Every time a future expires we “roll-over” to the next futures contract that is next to expire.

Let us now have a quick look (see the figure below) at the performance of the Greek stock market index during the last several years (blue line) versus that of the Greek banking sector (red line). The systemic nature of the Greek Banks is more than obvious since we can see that the two indices move together, at least historically. We can see that a naïve investment strategy of a simple buy and hold portfolio on the Greek banks would generate significant losses for an investor. And this is quite important since many financial institutions as well as other companies were highly exposed to the Greek Banks that suffered these severe losses.

Figure 1 – performance of FTSE and FTSE-Banking indices



The co-movements of these two indices imply significant correlations between the FTSE and the FTSE-Banking, a fact that justifies our choice to use index futures as a hedging instrument. Since we cannot achieve a perfect hedge and because the effectiveness of a hedging strategy improves as the correlations between the prices of the asset and the hedging instrument increase, we choose the index futures because we expect that they will be able to provide a sufficiently effective hedge and in this way reduce basis risk.

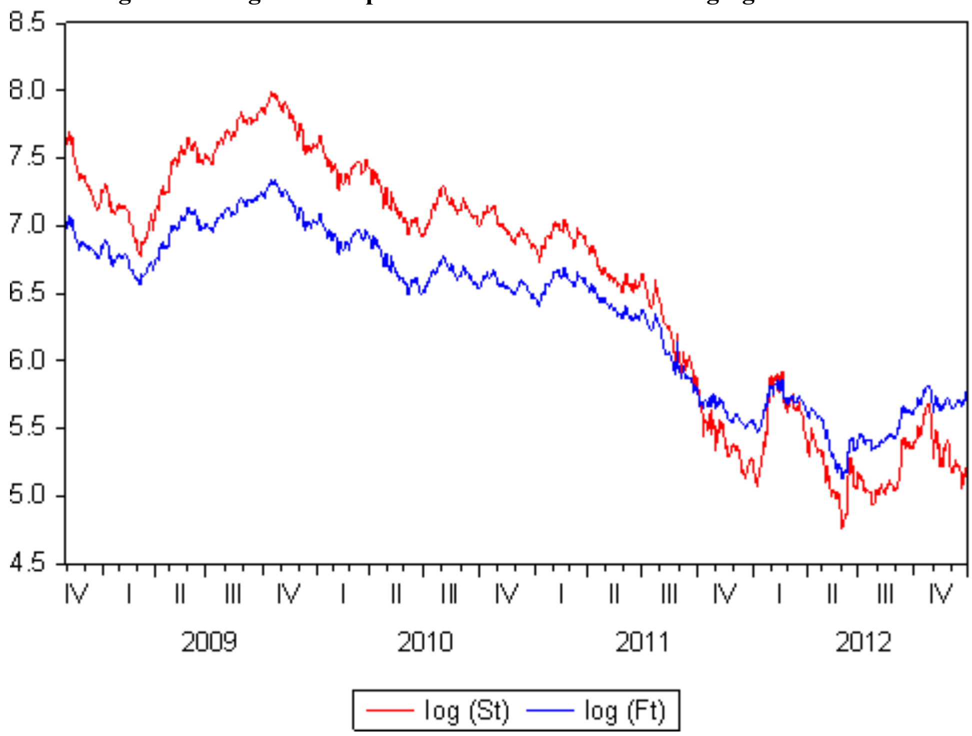
We see that during the last quarter of 2008 the market was falling, highly affecting the banking sector, which was falling much faster than the market. But, after 2008 the Greek stock market started rising again. By the end of 2009 the FTSE index reached approximately the 1,500 basis points. The banking sector was increasing rapidly at the same time, peaking almost at 3,000 basis points. When the bubble started to deflate, the crisis affected immediately the stock market and both indices started shrinking in a very fast way. By the end of 2012 the FTSE large cap index had lost almost 80% of its value in less than 3 years. The case of the banking index is even worse since we can see that this index was declining even more rapidly compared to the FTSE, showing that the banking sector suffered from losses of more than 90% during the same 3-year period (2010-2012).

Data Analysis

Statistical Properties

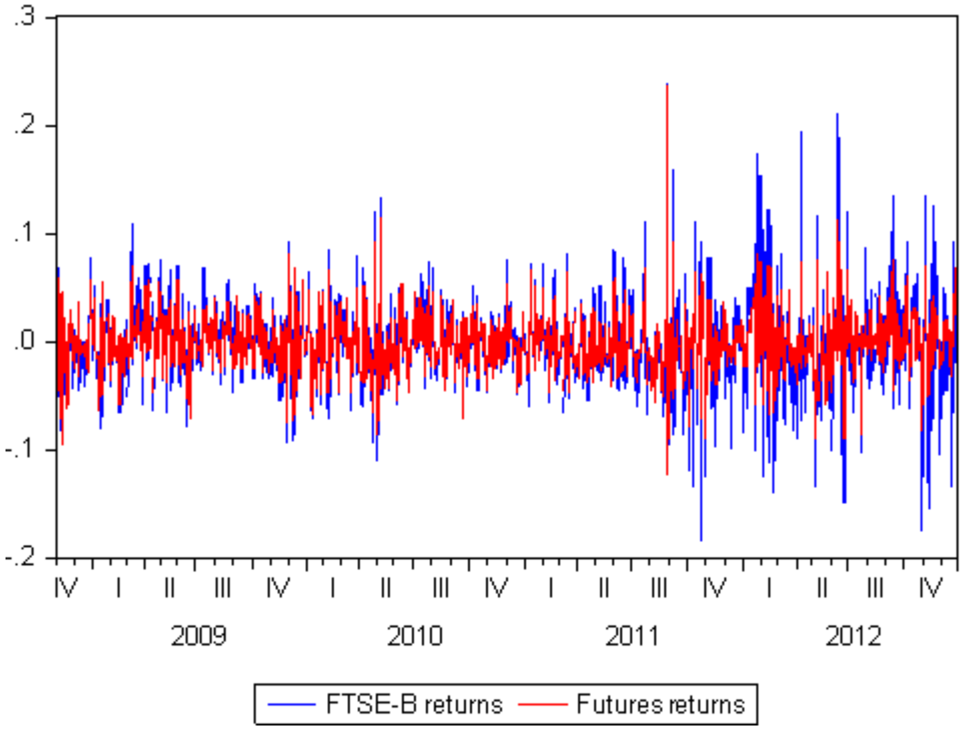
At this point it would be important to have a quick glimpse at the statistical properties of the data series that we are using. This is crucial in order to ensure that the methodological approach that we are using is valid. We will be using logarithmic prices in order to estimate the logarithmic returns. In order to make the notation more general we will use S_t for the spot price of the asset that is to be hedged and F_t for the futures prices that we use as a hedging instrument. The logarithmic returns for an asset are defined as $r_t^S = d\log(S_t) = \log(\frac{S_t}{S_{t-1}})$ and similarly for the futures prices. The following figure shows the evolution of the logarithmic levels of the spot prices of the asset (red line) and the hedging instrument (blue line). In this case we consider that the asset that is to be hedged is the FTSE Banking Index and the hedging instrument is the futures contract on the FTSE large cap index.

Figure 2 – Logarithmic prices of the asset and the hedging instrument



It is also important to focus on the statistical properties of the returns as well. The following figure shows how the logarithmic returns evolve in time. We can see that the returns are centralized around a mean value that is close to zero, which is normal. There are some periods of higher volatility, especially after the second quarter of 2011. This is a strong indication of volatility clustering in the data and it coincides with the period when the market was falling rapidly. This is a common stylized fact in financial data series since it is very often that volatilities increase when the markets are declining. There are also some outliers, i.e. some extreme returns that will somehow affect the econometric analysis.

Figure 3 – Returns of the Banking index and the FTSE index futures



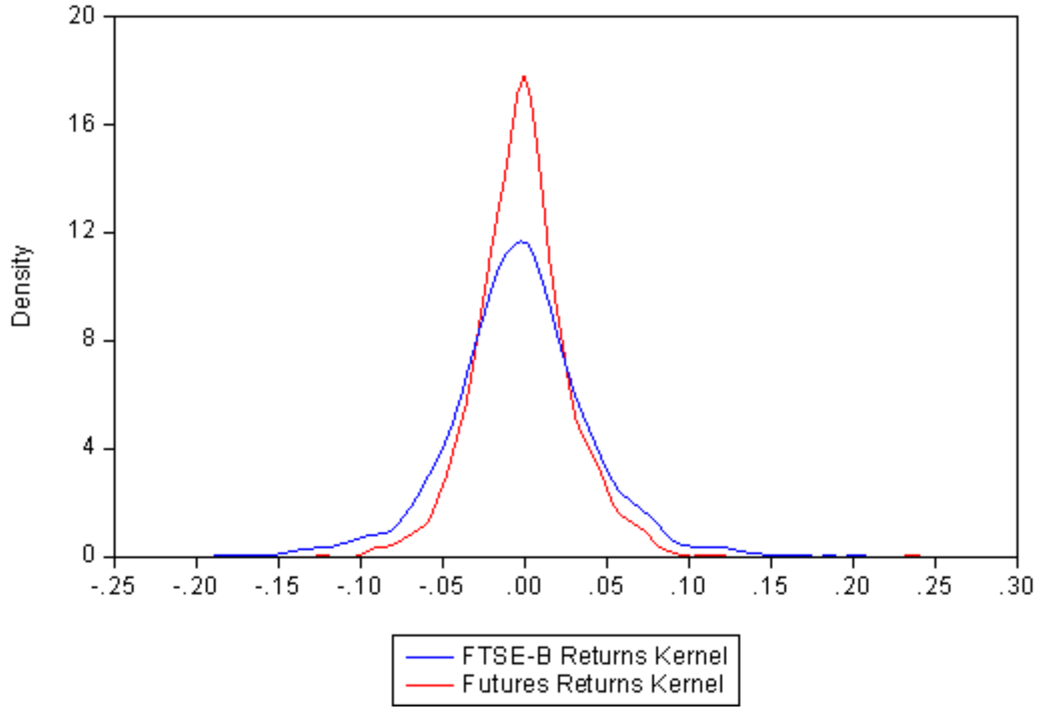
In the following table we have summarized some of the most important statistical properties of the logarithmic returns. We can see that the mean and median returns are close to zero and slightly negative. For instance, the FTSE-B index was declining by 0.24% on a daily basis, with a daily volatility of 4.43%. The futures prices were also declining, but the mean return is 0.12% daily and the volatility is 2.96%.

Table 1 – Descriptive Statistics of the asset and the instrument

	FTSE-B Returns	Futures Returns
Mean	-0.002374	-0.001191
Median	-0.003200	-0.001547
Maximum	0.239107	0.237017
Minimum	-0.183841	-0.123061
Std. Dev.	0.044368	0.029584
Skewness	0.291653	0.592309
Kurtosis	5.907848	7.764467

There is an observable asymmetry in the distributions of the two time series. We can see that both the FTSE-B and the futures returns have a small positive skewness, which means that the probability of getting a return that is higher than the mean is slightly greater than 50%. Both distributions have an excess kurtosis, since the level of kurtosis is higher than 3, which is the normal value for a standard-normal distribution. This indicates that a higher probability is gathered in the middle and in the tails of the distributions. This can also be seen from the kernel density plot below.

Figure 4 – Density kernels of the FTSE-B and the FTSE futures returns



Another important property that we need to test is the stationarity of the data. As we have already mentioned in the previous chapter, it is quite normal for financial time series that the prices are non-stationary processes, but this is not the case for the returns. This means that normally price levels are integrated of first order, or first-order stationary. In order to test this property we need to apply a unit-root test in the time series of the prices and the returns. We will use the Augmented Dickey Fuller test in order to examine the existence of a unit root in the data. Results are presented in the table that follows.

Table 2 – Augmented Dickey Fuller tests for unit root in logarithmic levels and first differences

	FTSE-B log-prices	Futures log-prices	FTSE-B Returns	Futures Returns
ADF test statistics	-0.3358	-0.7786	-31.1843	-25.1628
p-values	0.9170	0.8242	0.00*	0.00*

Test critical values for one-sided tests (MacKinnon, 1996):
1% level: -3.4364
5% level: -2.8641

The ADF test reveal that there is indeed a unit root in the log-levels of both time-series, since the null hypothesis of unit root cannot be rejected by the test. It is easy to see that that the levels are non-stationary if we compare either the ADF t-statistic with the MacKinnon critical values or the corresponding p-values with the required level of statistical significance. Since the p-values of the ADF test statistics for the returns of both FTSE-B index and futures time series are almost zero, this means that the returns series are stationary. In other words the levels are $I(1)$ since their first differences can produce a stationary process.

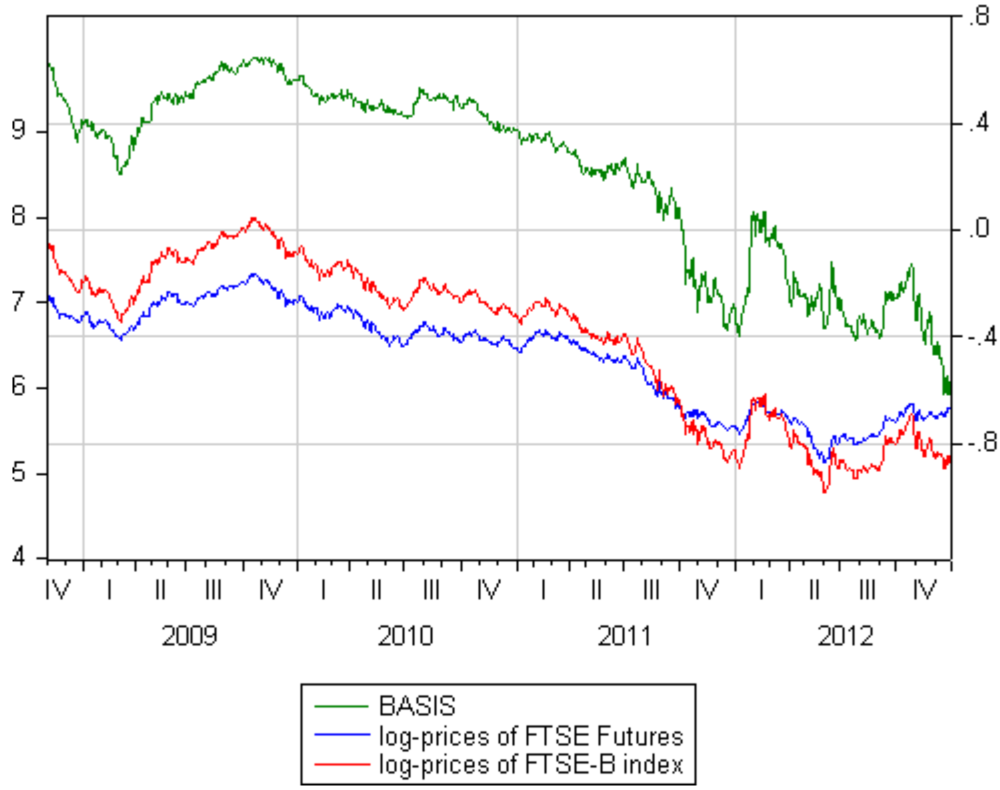
At this point we should underline that since both series are $I(1)$ this means that there is a linear combination between the levels of the two series that can produce a stationary process. We say that the two variables are co-integrated and thus we could alternatively use an error-correction model in order to estimate the hedge-ratio. In order to do it we could run a regression in levels that would have the form: $\log(S_t) = a + \beta \log(F_t) + \epsilon_t$ and capture the hedge ratio from the slope of the trend line. This implies that there is a long-run equilibrium between the log-levels of S_t and F_t and any deviation from this long-run trend should only survive for a short period of time. The error-correction mechanism (ECM) should drive these deviations back to the equilibrium in order to ensure the stationarity of the residual term.

As we have seen there are many studies in the literature that focus on error-correction models in order to estimate the hedge ratios, but these studies produce very similar results to the classical OLS regression models. For this reason, we have chosen to run a regression in the returns, since they are stationary $I(0)$, ignoring any possible cointegration relations in the levels of the data.

Basis Risk

In order to make the differences between the logarithmic prices of the asset and the hedging instrument more observable we estimate $B_t = \log(S_t) - \log(F_t)$, which is actually the way to measure basis risk. These are exactly the differences that introduce risk during hedging process that is caused by the imperfectness of the hedge. Since the futures and the asset that is to be hedged do not exactly match, there is a residual risk that cannot be hedged away by the position in futures. In the following figure we present the basis in the secondary axis with respect to the logarithmic prices of the FTSE-B index and the FTSE futures in the primary axis. What we could say is that as the basis comes closer to zero, the hedging becomes more effective. We see that the basis has been positive since the end of 2011, but after 2011 it turns negative because the banking sector had lost a significant percentage of its value.

Figure 5 – Basis Risk



Analysis of the Results

At this point, and after explaining all the methodological details of the approach, it is interesting to see how the procedure would be applied to real data. To estimate the optimal hedge ratio we can simply regress the logarithmic returns of the spot prices of the asset that needs to be hedged on the returns of the futures contracts that we use as a hedging instrument:

$$d\log(S_t) = a + \beta\, d\log(F_t) + \epsilon_t$$

For our econometric analysis we have chosen to use Eviews7. Results of the Ordinary Least Squares (OLS) method are summarized in the following table:

Table 3 – Ordinary Least Squares estimation results

Dependent Variable: FTSEB_RETURNS
Method: Least Squares
Date: 11/01/13 Time: 20:46
Sample (adjusted): 11/03/2008 12/31/2012
Included observations: 1041 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000773	0.000610	-1.266914	0.2055
FUTURES_RETURNS	1.344366	0.020623	65.18729	0.0000

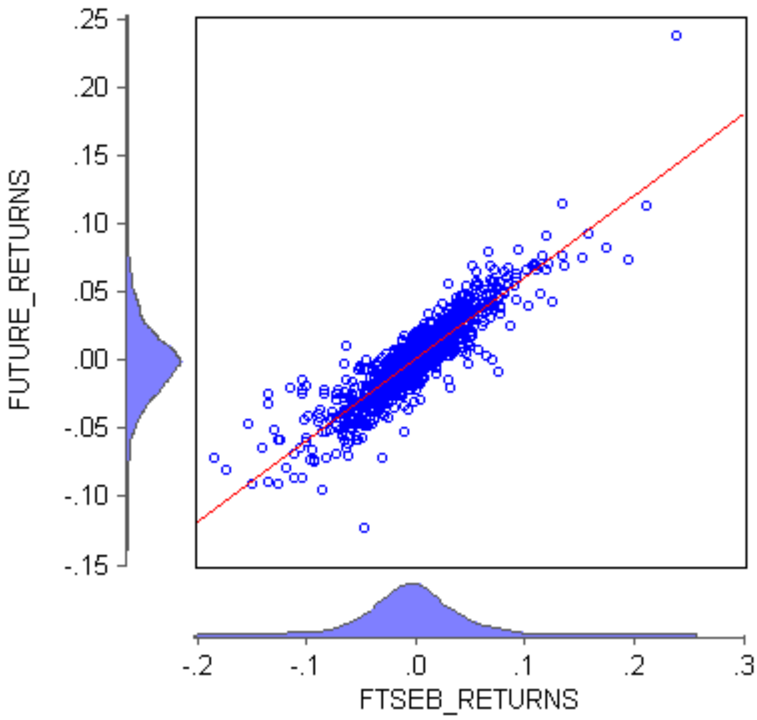
R-squared	0.803532	Mean dependent var	-0.002374
Adjusted R-squared	0.803343	S.D. dependent var	0.044368
S.E. of regression	0.019675	Akaike info criterion	-5.016975
Sum squared resid	0.402219	Schwarz criterion	-5.007469
Log likelihood	2613.336	Hannan-Quinn criter.	-5.013369
F-statistic	4249.383	Durbin-Watson stat	2.137709
Prob(F-statistic)	0.000000		

It is important to interpret these results properly in order to draw the correct information. As we have already explained in the Methodology section, the beta coefficient of this regression is exactly equal to the optimal hedge ratio that we wanted to estimate. This is the optimal positioning in futures in order to minimize the variance of the portfolio. We see that the optimal hedge ratio is $h^* = 1.34$, which implies that for every unit of asset that we include in the portfolio, we need to take a short position in futures that is 1.34 times the magnitude of the positioning in the spot asset value.

To be more specific, let us consider an investor that was exposed to the Greek Banking sector by having a position of 100,000 euros in a weighted portfolio of stocks of Greek Banks similar to the composition of the FTSE-Banking index. In order to hedge this position with short-term index futures of the FTSE large-cap index, the investor should take the opposite position by shorting futures contracts that correspond to 134,437 euros of the underlying asset. If each contract refers to 100euro of the underlying this means that the investor should short 1344 contracts in total in order to avoid any variance in the value of his portfolio.

What we can also see from the results above is that this hedge ratio, i.e. the beta of the regression, is statistically significant with a very high t-statistic that corresponds to a p-value that is almost zero. On the other hand the constant term of the regression takes a value that is very close to zero and is also statistically insignificant. These statistical findings are in line with the literature on estimating optimal hedge ratios, not only with the linear regression models but also with other approaches as well.

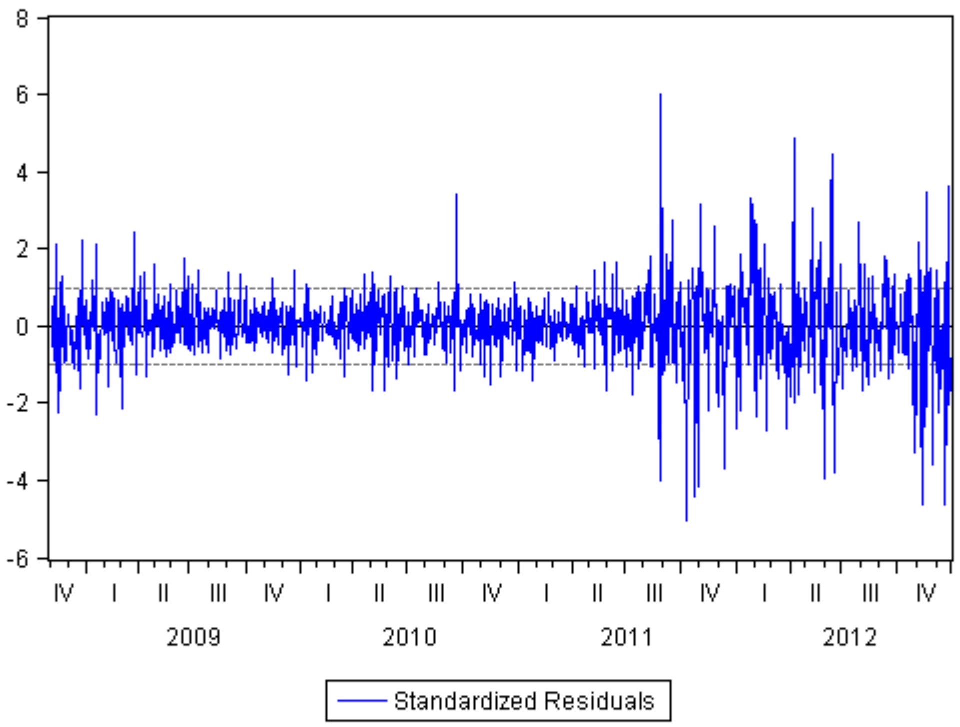
Figure 6 – Scatter plot of data around the theoretical regression line



The scatter plot above demonstrates the dispersion of the spot returns of the asset, which is the dependent variable, with respect to the returns of the futures prices, which is the independent variable in our regression model. We can see that a big part of the variation of the dependent variable can be explained by the variation in the explanatory variable. In order to see the explanatory power of the regression we need to focus on the coefficient of determination R^2, which is approximately 80%. As we have already explained, this coefficient equals $R^2 = \beta^2 \frac{\sigma_F^2}{\sigma_S^2}$, which can be written as $\rho^2 = h^{*2} \frac{\sigma_F^2}{\sigma_S^2}$. This is the way to quantify the effectiveness of the hedge. So the intuition behind this number is that only the 80% of the total variation of the spot price of the banking index can be hedged effectively with the use of FTSE index futures as a hedging instrument. The remaining percentage of the total variation cannot be hedged away due to the basis risk, showing the imperfectness of the hedging strategy. Alternatively, we could estimate the effectiveness of the hedge from the squared correlation coefficient between the two data series r^S and r^F, since we have already shown that $\rho = \frac{\sigma_{S,F}}{\sigma_S \sigma_F} = h^* \frac{\sigma_F}{\sigma_S}$.

It is also useful to take a closer look to the properties of the residual term of the linear regression. On the plot below we demonstrate the standardized residuals of the regression.

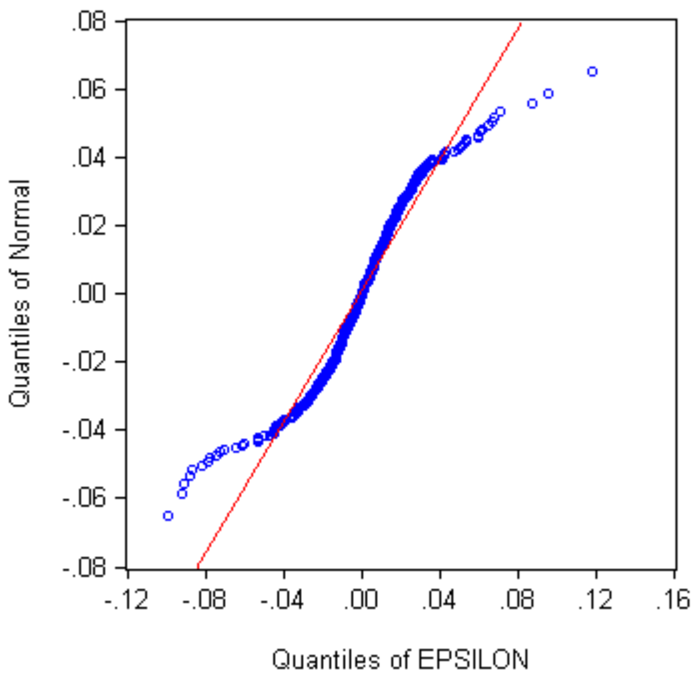
Figure 7 – Standardized Residuals of the regression and 95% Confidence Interval levels



We see that the error terms normally lie within the band that is determined by the 95% confidence interval level. There are some deviations that exceed the confidence bands and this is more intense after the third quarter of 2011. Maybe this is an indication that the hedging strategy started becoming less effective more lately, probably because the correlation of the two time series started to decline, hence affecting the hedging performance. We can also see this from the graph of the logarithmic levels of the two series. While the two series were moving with almost the same pattern until 2011, after the third quarter the banking index started being more volatile than the FTSE futures prices.

The descriptive statistics of the residual series show that the error term is centralized around zero, which is in line with the requirements of the linear regression models. The standard deviation is 1,97%. The errors seem to deviate from normality, since their distribution is negatively skewed with a skewness equal to -0.24 and has a kurtosis of 8.51, which is much higher than the normal. It is easy for one to see that the residuals deviate from normality if he compares the empirical quantiles of the error term with the theoretical quantiles of a standard normal distribution in the following QQ-plot.

Figure 8 – QQ-plot of the empirical dispersion of the residuals versus the theoretical standard normal quantiles



The last thing that we need to test in the residual term is the stationarity. It is very important for the error term of the regression to be stationary in order for the results to be valild. The ADF test reveals a very high t-statistic equal to -34.52, that corresponds to a p-value of zero. This means that the null hypothesis of a unit root in the residuals is strongly rejected, i.e. the series is a stationary process. This is to verify that the interpretation of the results is valid and not spurious.

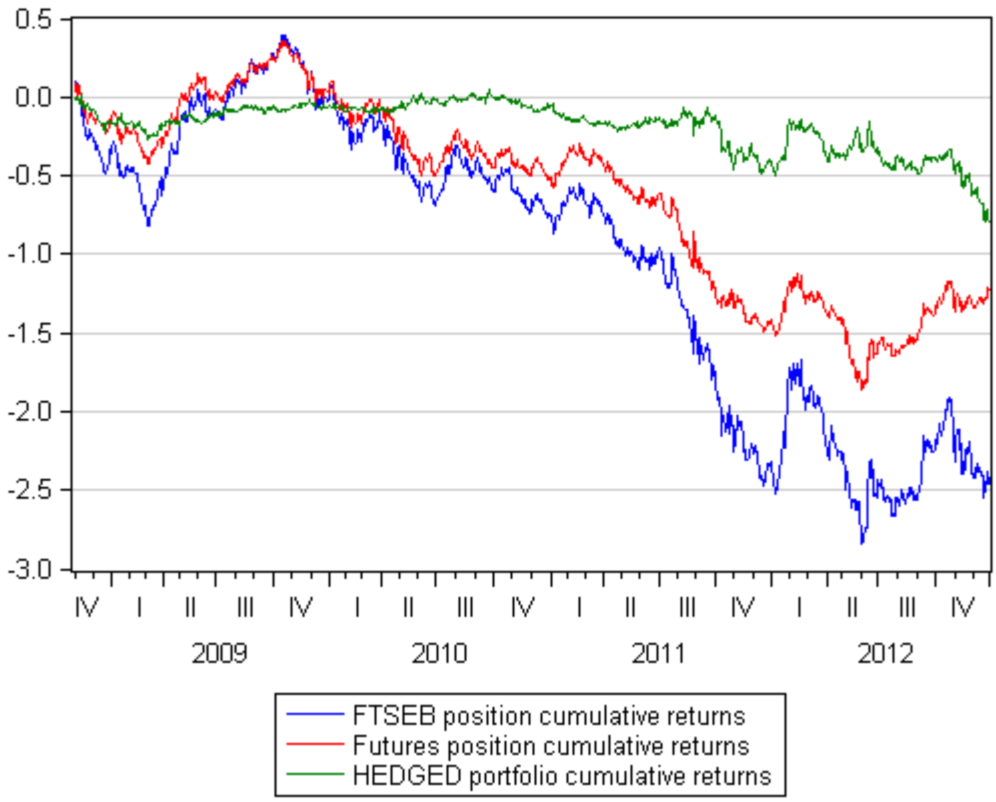
This econometric analysis that we have described so far is easy to implement and has interesting economic intuition. Even though we have focused on a specific time interval, this analysis could be very easily extended to other periods of time, other stocks or indices and different hedging instruments as well. More advanced econometric procedures could be potentially applied in order to capture the time-varying correlations among the assets and the instrument that is used to implement the hedge. The time-varying nature of the correlations is very important since it affects optimal hedge ratios and the hence the composition of the hedging portfolio.

Let us now finish our analysis by having a closer look to the economic aspect of the minimum-variance hedging for a risk-averse investor. Consider the investor that had a long position in the Greek banking sector, starting from October 2008. If he maintained this position he would suffer significant losses due to the financial crisis that highly affected the Greek banks. In the following figure we demonstrate the cumulative returns of each portfolio. As we can see, the naïve buy and hold strategy of a weighed portfolio of Greek banks would generate a cumulative return of -250% by the end of 2012 (The cumulative returns of the FTSE-B index are presented in blue color).

On the other hand, the prices of the short-term maturing FTSE large cap index futures also declined significantly during the same period. Thus, a naïve long position in index futures would also produce losses for an investor. The red line shows the cumulative returns for the long position in futures.

But, suppose that the investor calculates the optimal hedge ratio in order to minimize the total variance of his portfolio. First of all, he would like to maintain his long position to the Greek banks, at the same time building an opposite position with index futures. The minimum-variance hedge ratio that we estimated before showed that the optimal analogy for index futures is 1.34 contracts per unit of the asset that is to be hedged. Hence the investor constructs a hedging portfolio by going short 1.34 units of futures contracts per unit of the asset that he wants to hedge. He actually weights the two unhedged positions in the asset and the futures in order to construct a hedged position. A green line represents the cumulative returns on the hedged portfolio.

Figure 9 – Cumulative returns of Hedged portfolio versus cumulative returns of the long positions in the FTSE-B and the index futures



What we can see is that the cumulative returns of the hedged portfolio are quite close to zero, which actually verifies the concept of hedging in a portfolio. The cumulative returns to the hedged portfolio are much less volatile than those of the banking index or the futures, separately. By combining these two opposite positions in one portfolio the investor can avoid a significant part of the variation of the asset that he wants to go long. In this way he cannot participate in the gains of the banking sector during 2009. But if he were afraid of a possible collapse, the hedging position would save him from severe losses by making his position immune to either positive or negative market movements. It is important to underline that after 2009 when the Greek stock market and the banking index started to fall rapidly in value, the hedged portfolio always outperforms the naïve buy and hold strategy of a long exposure to the Greek banks.

The closer the cumulative returns of the hedged portfolio are to zero, the higher the effectiveness of the hedging strategy that we have constructed. The static approach of a simple linear regression model seems to perform quite well, especially until the end of 2011. After this point there is a small decrease in the effectiveness of the hedge that is due to the changes in the correlations between the FTSE-B index and the hedging instrument as we have already explained. This period coincides with the period of higher volatility for the banking sector and the increases in the basis risk that we have shown above. This also makes the returns of the hedged portfolio to be more volatile as well, since the basis risk increases and the imperfectness of the hedge becomes more pronounced. But overall, our results show that there is an effectiveness of 80% for an investor that hedges his exposure to the Greek banking system with FTSE index futures, and this percentage is quite sufficient in order to avoid the huge variation of the Greek stock market and the unexpected significant losses of the Greek banks.

Concluding Remarks

We have indeed experienced a period of extreme financial instability in Greece during the last several years. The deflation of the bubble and the crisis of the Greek stock market highly affected the Greek Banking sector. The banks started to shrink very rapidly and the investors that were exposed to this system suffered severe losses. This financial crisis initially was global and also contagious across the financial systems of different countries, and this was mainly due to the rapid expansion of the traded volume of financial derivatives that actually "boosted" the banking sector. Since banks hold positions of very high risk in their portfolios, this risk was soon translated into systematic risk for the total economy, and this is because banks had become big enough to drive financial markets. The case in Greece was even more pronounced, since the Greek Banking sector comprises the biggest component of the major Greek stock market index.

The aim of this dissertation was to study the hedging performance of an investor that maintained an exposure to the Greek banking sector. We focus on the period of the recent financial crisis because it is very challenging to study how a naïve hedging strategy would help to avoid the severe losses of the Greek banking sector. During this period the Greek stock market was very volatile, accompanied by extreme losses that were caused by the systematic economic shocks transmitted via the banking system.

We have shown how the use of FTSE large-cap index futures contracts would allow an investor to hedge his exposure to the Greek banking sector. In our analysis, we consider an investor that has a long position into a portfolio of bank stocks and at the same time maintains a short position into stock index futures that are expected to expire soon. Since the returns of the Greek banking sector are strongly correlated with the returns of the total market index, this hedging strategy should produce sufficiently good results since it minimizes the basis risk.

We implement a static minimum-variance hedging approach that is estimated within a linear regression framework. Our primary interest is the intuition behind the minimum-variance hedging and its real-world application for a Greek investor and not the complexity of the estimation procedure.

Our results reveal that an investor can hedge himself from a significant proportion of the total price variation of the banking index by shorting an optimal amount of index futures. To be more specific, the optimal hedge ratio turns out to be 1.34 units of index futures per unit of the asset that is to be hedged, in this way providing a hedging effectiveness of 80% for the investor. We have observed that especially during the downward movement of the stock market, the hedging portfolio significantly outperforms the buy-and-hold strategy on the Banking index. The variability of the returns to the hedging portfolio is very small, indicating the effectiveness of the hedging strategy. Only during the last several months of our sample there is an increase in the volatility of the hedged returns. This is explained by the higher volatility of the banking index during the same period. The FTSE-B index started moving more intensely than the FTSE index and hence these two indices started becoming less correlated. This decline in the correlation introduces a higher basis risk in the hedging strategy and makes the imperfectness of the hedge slightly more pronounced.

This analysis could be further extended to capture the time-varying correlations between the asset and the hedging instrument. More advanced econometric procedures, like the GARCH approach, could potentially give us an indication about the time-varying nature of the optimal hedge ratios. A dynamic hedging strategy could possibly provide improved results compared to our naïve static hedging approach. There are numerous studies in the literature that focus on time-varying dynamic hedge ratios. However, this is beyond the scope of this dissertation but could be an area of further research.

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